Efficiency and stability in electrical power transmission networks

A partition function form approach

Dávid Csercsik[1] László Á. Kóczy[1 2]

koczy@krtk.mta.hu

[1]Research Centre for Economic and Regional Studies, Hungarian Academy of Sciences

[2]Óbuda University

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Project outline

Project objectives

Study the stability of cooperative games with

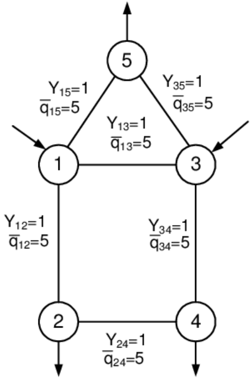
- network structure &
- widespread externalities.

1. Coalition formation game over an exogenous network. (Here, illustrated by a power network)
2. Endogenous coalition & network formation games. (Work in progress)

Electric networks

An electric network consists of

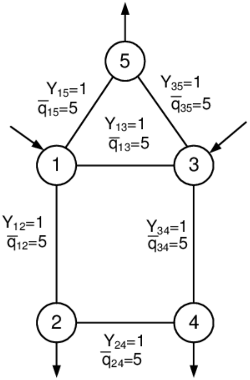
- Nodes: generators and consumers
- Arcs: power lines



Electric networks

An electric network consists of

- Nodes: generators and consumers
  - With production/consumption capacity constraints.
- Arcs: power lines with
  - transmission capacity constraints
  - given susceptance[1] levels



[1] The imaginary part of admittance, the "opposite" of resistance.

Managing electric networks

- FACT 1: difficult to store electricity

$\therefore$ Production = consumption

- FACT 2: costly to reschedule generators (to meet demand)

$\therefore$ *Balancing groups*

$\Rightarrow$ A problem of coalition formation.

- FACT 3: Electricity routes itself ... based on network characteristics
- Operator sets in/output to avoid overload ... on the fly.
- Balancing groups simplify the task.

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Our model

PART I: simplified electric flows

- Capacity and susceptance of power lines is given
- Flow is lossless.
- Operator maximizes total production/consumption

PART II: group formation

- Partition into balancing groups
- Additional constraints for LP problem
- Coalition formation problem.

Our model formally

n nodes, m arcs among them.

- $A \in \mathbb{R}^{n\times m}$ incidence matrix
- $B \in \mathbb{R}^{n\times n}$ susceptance matrix (=imaginary part of admittance)
- $B^D \in \mathbb{R}^{m\times m}$ is a diagonal matrix of suspceptance values.
- $P \in \mathbb{R}^n$ power injection vector
- $Q \in \mathbb{R}^m$ real flow vector
- $\bar{Q} \in \mathbb{R}^m$ is the vector of capacities

$$AP = Q$$

$$\min_P s_P^T P \text{ s.t. } \left| B^D A^T B^+ P \right| < \bar{Q}, \mathbf{1}P = 0$$

+conditions for the coalitions
(s_P is a sign vector, B^+ is B's Moore-Penrose pseudoinverse.)

Partition function

- Solution of LP problem gives optimal flows.

$\Rightarrow$ Well-defined production/consumption for nodes

$\Rightarrow$ Well-defined utility for nodes

- Allow TU, so coalitional payoffs
- ...*given* a partition into balancing groups.

Repeat for all partitions

$\Rightarrow$ Game in partition function form $V : \Pi \rightarrow (2^N \rightarrow \mathbb{R})$.

Our goals

- Better understand these networks
- Study properties, namely:
  - Nature of externalities
  - Superadditivity
  - Stability (using the *recursive core*)
  - Etc. ($\rightarrow$ future plans)

Negative results using counterexamples...

Externalities

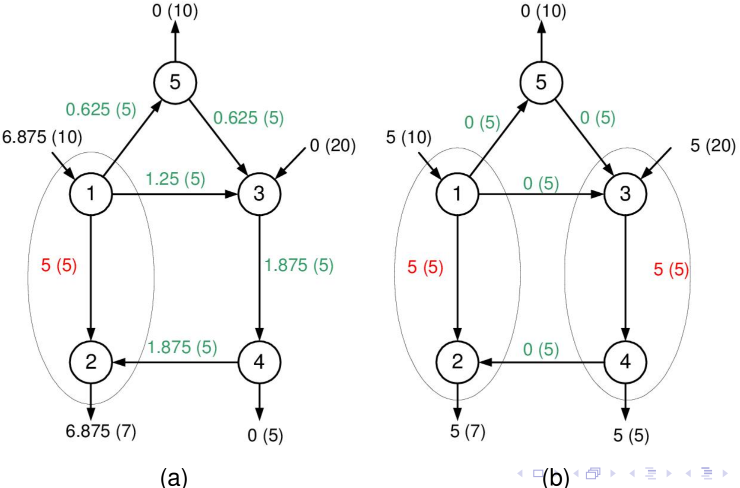
Negative externalities

The payoff of outside coalitions weakly decreases after a merger of two or more coalitions

- Negative externalities are expected:
  - Merged coalition removes some LP constraints
  - New trading possibilities
  - Additional load on the network
  - Others can put less load due to capacity constraints.
- BUT: example with positive externalities

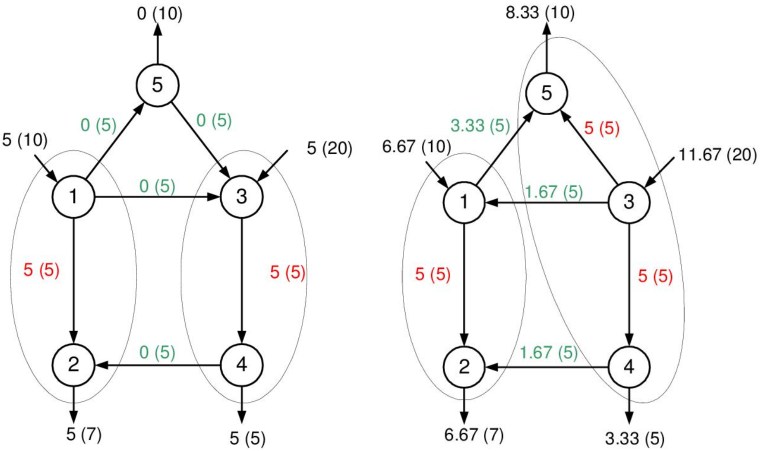
Externalities – the usual case

Figure : Payoff of $\{1, 2\}$ goes down.



Positive externalities

Figure : Payoff of $\{1,2\}$ goes up.



(a) (b)

Superadditivity

Superadditivity

The total payoff of merged coalitions is weakly higher.

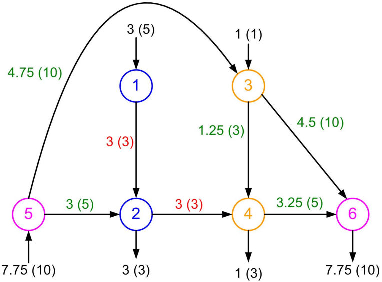
- The merger removes a constraint in the LP problem.
- (Flows are not optimised for *this* coalition!)
- Total payoff is higher.
- Usually externalities are negative

∴ *Usually* we should have superadditivity

- Externalities *may* be positive
- *Externalities be higher than the gains from the merger.*

A case of subadditivity

Figure : The merger of $\{1, 2\}$ and $\{3, 4\}$ leads to losses.



(a) (b)

Cores

3 types of cores are used:

α-core (Aumann & Peleg, Bulletin of the AMS, 1960)

A deviating coalition expects the worst partition. $\therefore$ Core is small.

Core stability (Shenoy, IJGT, 1979)

A deviating coalition expects the best partition. $\therefore$ Core is large.

Recursive core (Kóczy, Theory & Decision, 2007)

1. A deviation induces a residual subgame
2. A deviating coalition expects a core (if empty: any) partition in the corresponding residual subgame.
3. Optimistic/pessimistic versions often coincide.

Cores – properties?

- If the residual partition is

  all singletons: no load on the network, but poor trading possibilities

  grand coalition: loaded network, good trading possibilities

  in between: ???
- Difficult to establish general results.
- The residual core can be empty

Cores – 5 player example

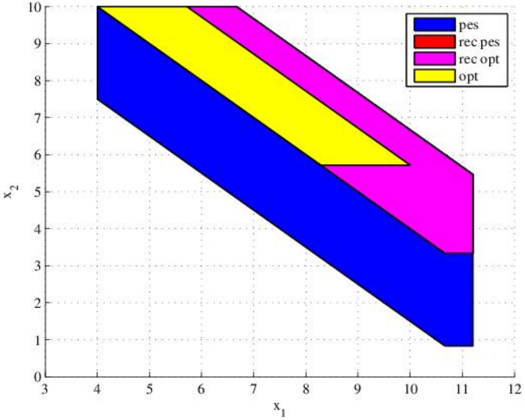


Figure : The projection of the cores onto $x_3 = x_4 = x_5 = 0$. The recursive cores coincide, others in inclusive relation.

Cores – 6 player example

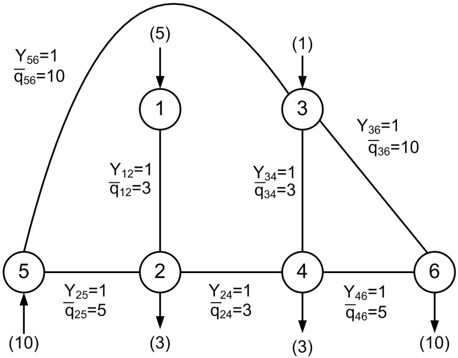


Figure : The core of this 6-player game is empty.

Related work

- Traffic routing with congestion (Csercsik & Sziklai)
  - a road network
  - players have routing tasks
  - players may coordinate tasks or share load information
    - Is this in their interest?

  A game of routeplanners. Routeplanners can predict and control congestion.
- Telecom routing (Csercsik & Imre)
  - a fixed telecommunication network
  - latency is due to the routers at the nodes
  - finite tasks with heterogeneous sensitivity to latency
  - compare router intelligent (send in the right direction) and host intelligent (HIR, plan entire route) routing

  HIR can reduce latency a bit, but increase utility a lot.

Related work II

- Coalitional stability in networks (Kóczy, work in progress)
  - players can form coalitions
  - coalitions can reorganise internal, break external links
  - coalitional payoff is a function of the network, the partition.
  - generalisation of the recursive core
  - study a favour network: costly relations, benefits from friends' friends
    - Empty core unless intermediaries are well compensated.

Network of flights.

Summary

- A cooperative game theoretic model of balancing group formation over a power network.
- The game is in partition function form.
- Externalities can be positive or negative.
- The game is not necessarily superadditive.
- The recursive core of such a game may be empty.

Future work/open issues

Extensions of the model:

- Seasonal or daily variation of demand.
- Risk of lines breaking down.
    - by measures of risk,
    - by games with uncertainty (Habis & Herings, JET, 2010)
- Strategic building/destruction of lines.